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TIME TO FAILURE AND RELIABILITY ANALYSIS OF SERIES PARALLEL COMPLEX SYSTEMMODELS WITH REPAIR

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ABSTRACT

In this paper, the authors deal with the reliability and time to failure of a complex system model. This complex system contains two subsystem A and B, Where system Acontains N units in series and system B contains M units in parallel. Failure of any one unit of A gives failure of subsystem A, hence the whole system fails but failure of one unit of B does not fail the whole system. We have assumed 2-out-of-n-general policy for B. It is assumed that all the failures follow exponential time distribution whereas the repairs follow general time distribution.

KEYWORDS: Markovianprocess, Supplementary Variable Technique, Laplace Transforms, Steady State Behavior, Exponential Time Distribution

INTRODUCTION

In the present paper we study acomplex system modeland obtain various reliability characteristics which are of interests to system designers and operation managers. We obtain these characteristics by using supplementary variable techniques. In our system, we have taken a general model transit system under head of line repair policy. The system contains two subsystems A and B. where system A contains N units in series and system B contains M units in parallel. By virtue of arrangements of units in the system, failure of any one unit of A results in the failure of the whole subsystem A and thereby breaking down the whole system, whereas the failure of any one unit of B does not result in the failure of the subsystem B and so the system continue to function. We have assumed 2-out-of-n-general policy for B i.e. if any 2 units in B fail then we find the degraded state and i9f more than 2 unit fail, it results in the failure of whole system. Hence system will fail if either any 1 of the units in A fails or if 3 or more units of B fail. The repair carried out only when the system breaks down and each repair makes the system as good as it was in original condition. All the failure time distributions are taken to be negative exponential while the repair time distributions are general. The transistor diagram of the system model is shown in figure 1.1.

ASSUMPTIONS

1. Initially, all components of the system are good.
2. The reliability of each components of the system is known in advance. In one state ,only one change can take place.
3. The state of each components of the system is either good or bad.
4. The state of all components is independent. The failure times of all components are Statistical independent.
5. After repair the system works as good as new.

NOTATIONS AND STATES OF THE SYSTEMS

$P_N^m(t) \mid P_N^{m-1}(t)$: Probability that the system is in working state at time t when m/m-1 units of sub system B are in working state, respectively.

$P_N^{m-2}(t)$: probability that the system is in degraded state at time t due to failure of 2 units of subsystem B.

u_i : failure rate of i^{th} unit of subsystem A.

V_j: failure rate of j^{th} unit of subsystem B.

$U = \sum_{i=1}^{N} u_i$, $V_1 = \sum_{J=1}^{m} V_j$, $V_2 = \sum_{j=1}^{m-1} V_j$, $V_3 = \sum_{j=1}^{m-2} V_j$

$\alpha_i(x)$: Repair rate of i^{th} subsystem, h_i : Human error rate of failure.

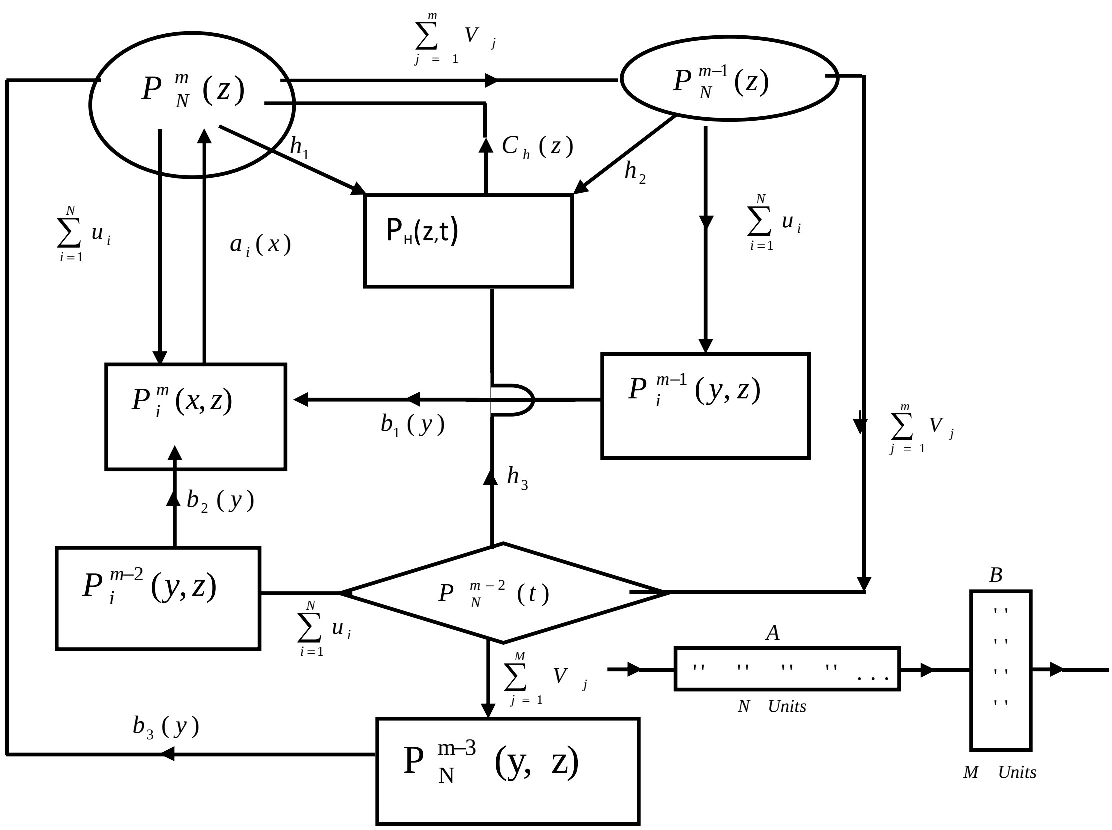

Figure 1.0: Transition Diagram

MATHEMATICAL FORMULATION OF THE MODEL

Simple probabilistic considerations and limiting procedure yields the following set of difference-differential equations for the stochastic process which is continuous in time and discrete in space.

Basic Equations

$$\left[\frac{d}{dz}+u+v_1+h_1\right] P_N^m(z)=\sum_{i=1}^{N}\int_0^\infty P_i^m(x,z)a_i(x)dx+\int_0^\infty P_N^{m-3}(y,z)b_3(y)dy+\int_0^\infty P_N(z,t)c_h(z)dz \quad (1)$$

$$\left[\frac{d}{dz}+u+v_2+h_2\right]P_N^{m-1}(z)=v_1P_N^m(z) \quad \ldots\ldots \quad (2)$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial z}+a_i(x)\right]p_i^m(x,z)=0 \quad \ldots\ldots \quad (3)$$

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial z}+b_1(y)\right]p_i^{m-1}(y,z)=0 \quad \ldots\ldots \quad (4)$$

$$\left[\frac{d}{dz}+u+v_3+h_3\right] P_N^{m-2}(z)=\sqrt{2}P_N^{m-1}(z) \quad \ldots\ldots \quad (5)$$

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial z}+b_2(y)\right]P_i^{m-2}(y,z)=0 \quad \ldots\ldots \quad (6)$$

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial z}+b_3(y)\right]P_N^{m-3}(y,z)=0 \quad \ldots\ldots \quad (7)$$

$$\left[\frac{\partial}{\partial z}+\frac{\partial}{\partial z}+c_n(z)\right]P_H(z,z)=0 \quad \ldots\ldots \quad (8)$$

Boundary Conditions

$$P_i^m(0,z)=u\ P_N^m(z)+\int_0^\infty P_i^{m-1}(y,z)b_1(y)dy+\int_0^\infty P_i^{m-2}(y,z)b_2(y)dy \quad \ldots\ldots \quad (9)$$

$$P_i^{m-1}(0,z)=uP_N^{m-1}(z) \quad \ldots\ldots \quad (10)$$

$$P_i^{m-2}(0,z)=uP_N^{m-2}(z) \quad \ldots\ldots \quad (11)$$

$$P_i^{m-3}(0,z)=v_3P_N^{m-2}(z) \quad \ldots\ldots \quad (12)$$

$$P_N(0,z)=h_1P_N^m(z)+h_2P_N^{m-1}(z)+h_3P_N^{m-2}(z) \quad \ldots\ldots \quad (13)$$

Initial Condition: It is assumed that the system initially starts from

$P_N^m(0)=1$,otherwise zero. ……. (14)

SOLUTION OF THE MODEL

Taking Laplace Transform of the equations (1) through (13) and then, on solving them subjected to (14), we obtain:

$$[s+u+v_1+h_1]\overline{P}_N^m(s)=1+\sum_{i=1}^{N}\int_0^\infty \overline{P}_i^m(x,s)a_i(x)dx+\int_0^\infty \overline{P}_N^{m-3}(y,s)b_3(y)dy$$
$$+\int_0^\infty \overline{P}_N(z,s)c_h(z)dz \quad \ldots\ldots \quad (15)$$

$$[s+u+v_2+h_2]\overline{P}_N^{m-1}(s)=v_1\ \overline{P}_N^m(s) \quad \ldots\ldots \quad (16)$$

$$\left[\frac{\partial}{\partial x}+s+a_i(x)\right]\bar{P}_i^m(x,s)=0 \quad .... \quad (17)$$

$$\left[\frac{\partial}{\partial y}+s+b_1(y)\right]\bar{P}_i^{m-1}(y,s)=0 \quad .... \quad (18)$$

$$[s+u+v_3+h_3]\bar{P}_N^{m-2}(s)=v_2\bar{P}_N^{m-1}(s) \quad ...... \quad (19)$$

$$\left[\frac{\partial}{\partial y}+s+b_2(y)\right]\bar{P}_i^{m-2}(y,s)=0 \quad .... \quad (20)$$

$$\left[\frac{\partial}{\partial y}+s+b_3(y)\right]\bar{P}_i^{m-3}(y,s)=0 \quad .... \quad (21)$$

$$\left[\frac{\partial}{\partial z}+s+c_h(z)\right]\bar{P}_N(z,s)=0 \quad .... \quad (22)$$

$$\bar{P}_i^m(0,s)=u\bar{P}_N^m(s)+\int_0^\infty \bar{P}_i^{m-1}(\partial,s)b_1(y)dy+\int_0^\infty \bar{P}_i^{m-2}(\partial,s)b_2(y)dy \quad ...... \quad (23)$$

$$\bar{P}_i^{m-1}(0,s)=u\bar{P}_N^{m-1}(s) \quad .... \quad (24)$$

$$\bar{P}_i^{m-2}(0,s)=u\bar{P}_N^{m-2}(s) \quad .... \quad (25)$$

$$\bar{P}_i^{m-3}(0,s)=v_3\bar{P}_N^{m-2}(s) \quad .... \quad (26)$$

$$\bar{P}_H(0,s)=h_1\bar{P}_N^m(s)+h_2\bar{P}_N^{m-1}(s)+h_3\bar{P}_N^{m-2}(s) \quad ....(27)$$

Now from equations (16) &(18-22) we get

$$\bar{P}_N^{m-1}(s)=\frac{v_1\bar{P}_N^m(s)}{s+u+v_2+h_2} \quad .... \quad (28)$$

$$\bar{P}_i^{m-1}(s)=\frac{uv_1\bar{P}_N^m(s)}{s+u+v_2+h_2}D_1(s) \quad ...... \quad (29)$$

$$\bar{P}_N^{m-2}(s)=\frac{v_1v_2\bar{P}_N^m(s)}{(s+u+v_2+h_2)(s+u+v_3+h_3)} \quad ...... \quad (30)$$

$$\bar{P}_N^{m-2}(s)=\frac{u\ v_1v_2\ \bar{P}_N^m(s)}{(s+u+v_2+h_2)(s+u+v_3+h_3)}D_2(s) \quad .... \quad (31)$$

$$\bar{P}_N^{m-3}(s)=\frac{v_1v_2v_3\bar{P}_N^m(s)}{(s+u+v_2+h_2)(s+u+v_3+h_3)}D_3(s) \quad ...... \quad (32)$$

$$\bar{P}_N(s)=\bar{P}_N^m(s)\left[h_1+\frac{v_1}{s+u+v_2+h_2}\left\{h_2+\frac{h_3v_2}{s+u+v_3+h_3}\right\}\right]D_h(s) \quad (33)$$

Also from equation (17), we get

$$\bar{P}_i^m(s) = u \quad \bar{P}_N^m(s) \left[1 + \frac{v_1}{s+u+v_2+h_2} \left\{\bar{S}_1(s) + \frac{v_2\bar{S}_2(s)}{s+u+v_3+h_3}\right\}\right] D_i(s) \text{......} \quad (34)$$

Finally, $\bar{P}_N^m(s) = \frac{1}{A(s)}$ .... (35)

where

$$A(s) = s+u+v_1+h_1 - u\left[1 + \frac{v_1}{s+u+v_2+h_2}\{\bar{S}_1(s) + \frac{v_2\bar{S}_2(s)}{s+u+v_3+h_3}\right] \text{.......} \quad (35a)$$

$$\bar{S}_i(s) - \frac{v_1v_2v_3\bar{S}_3(s)}{(s+u+v_2+h_2)\ (s+u+v_3+h_3)} \left[h_1 + \frac{v_1}{s+u+v_2+h_2}\left\{h_2 + \frac{h_2v_2}{s+u+v_3+h_3}\right\}\right]\overline{S_h}(s) \text{....} \quad (36)$$

Now, Let

$$B(s) = \frac{v_1}{(s+u+v_2+h_2)} \ \& \ C(s) = \frac{v_2}{(s+u+v_3+h_3)} \text{ Then}$$

Finally, we may obtain

$$\bar{P}_N^m(s) = \frac{1}{A(s)} \text{......} \quad (37)$$

$$\bar{P}_N^{m-1}(s) = \frac{B(s)}{A(s)} \text{.......} \quad (38)$$

$$\bar{P}_i^{m-1}(s) = \frac{u\ B(s)}{A(s)} D_1(s) \text{......} \quad (39)$$

$$\bar{P}_N^{m-2}(s) = \frac{B(s)\ C(s)}{A(s)} \text{......} \quad (40)$$

$$\bar{P}_i^{m-2}(s) = \frac{u\ B(s)\ C(s)}{A(s)} D_2(s) \text{.......} \quad (41)$$

$$\bar{P}_N^{m-3}(s) = \frac{v_3B(s)C(s)}{A(s)} D_3(s) \text{......} \quad (42)$$

$$\bar{P}_H(s) = \frac{1}{A(s)}\left[h_1 + B(s) \ \{h_2 + h_3C(s)\}\right] D_h(s) \text{......} \quad (43)$$

$$\bar{P}_i^m(s) = \frac{u}{A(s)}\left[1 + B(s)\left\{\bar{S}_1(s) + \bar{S}_2(s)C(s)\right\}\right] D_i(s) \text{......} \quad .(44)$$

Where

$$A(s) = s+u+v_1+h_1 - u \ \left[1 + B(s)\left\{\bar{S}_1(s) + \bar{S}_2(s)C(s)\right\}\right]$$

$$\bar{S}_i(s) - v_3B(s)C(s)\bar{S}_3(s) - \left[h_1 + B(s)\left\{h_2 + h_3C(s)\right\}\right]\bar{S}_h(s) \text{.......} \quad (45)$$

SOME PARTICULAR CASES

Now if we consider the situation where repair follows exponential time distribution , then putting $\overline{S}_i(s) = \dfrac{\mu_i}{s+\mu_i}$ for all i and s in equations (37) gives.

$$\overline{P}_N^{m}(s) = \frac{1}{E(s)} \qquad \ldots\ldots\ (46)$$

$$\overline{P}_N^{m-1}(s) = \frac{B(s)}{E(s)} \qquad \ldots\ldots\ (47)$$

$$\overline{P}_i^{m-1}(s) = \frac{u\ \ B(s)}{E(s)} \cdot \frac{1}{s+b_1} \qquad \ldots\ldots\ (48)$$

$$\overline{P}_N^{m-2}(s) = \frac{B(s).C(s)}{E(s)} \qquad \ldots\ldots\ (49)$$

$$\overline{P}_i^{m-2}(s) = \frac{uB(s)C(s)}{E(s)} \cdot \frac{1}{s+b_2} \qquad \ldots\ldots\ (50)$$

$$\overline{P}_N^{m-3}(s) = \frac{u_3B(s)C(s)}{E(s)} \cdot \frac{1}{s+b_3} \qquad \ldots\ldots\ (51)$$

$$\overline{P}_N(s) = \frac{1}{E(s)}\left[h_1 + B(s)\{h_2 + h_3C(s)\}\right]\frac{1}{s+C_h} \qquad \ldots\ldots\ (52)$$

$$\overline{P}_i^{m}(s) = \frac{u}{E(s)}\left[1 + B(s)\left\{\frac{b_1}{s+b_1} + \frac{b_2\ \ c(s)}{s+b_2}\right\}\right]\frac{1}{s+ai} \qquad \ldots\ldots\ (53)$$

Where

$$E(s) = s + u + v_1 + h_1 - u\left[1 + B(s)\left\{\frac{b_1}{s+b_1} + \frac{b_2c(s)}{s+b_2}\right\}\right]\frac{a_i}{s+a_i} - v_3B(s)c(s)\frac{b_3}{s+b_3}$$

$$\left[h_1 + B(s)\ \ \{h_2 + h_3c(s)\}\right]\frac{c_h}{s+c_h} \qquad \ldots\ldots\ (54)$$

ERGODIC BEHAVIOUR OF THE SYSTEMS

By making use of Abel's Lemma $\lim_{S\to 0} \overline{P}(s) = \lim_{t\to\infty} P(t) = P$ (say)(Provided the limit on right exists) , we have the following time independent state probabilities from equations (37) through (44).

$$P_N^{m} = \frac{1}{A'(0)} \qquad \ldots\ldots\ (55)$$

$$P_N^{m-1} = \frac{1}{A'(0)} \cdot \frac{v_1}{u + v_2 + h_2} \qquad \ldots\ldots\ (56)$$

$$P_i^{m-1} = \frac{u}{A'(0)} \cdot \frac{v_1}{u + v_2 + h_2}.m_1 \qquad \ldots\ldots. \qquad (57)$$

$$P_N^{m-2} = \frac{1}{A'(0)} \cdot \frac{v_1 \quad v_2}{(u + v_2 + h_2)(u + v_2 + h_3)} \qquad \ldots\ldots \qquad (58)$$

$$P_i^{m-2} = \frac{u}{A'(0)} \cdot \frac{v_1 \quad v_2}{(u + v_2 + h_2)(u + v_2 + h_3)} m_2 \qquad \ldots\ldots. \qquad (59)$$

$$P_N^{m-3} = \frac{1}{A'(0)} \cdot \frac{v_1 \quad v_2 \quad v_3}{(u + v_2 + h_2)(u + v_3 + h_3)} m_3 \qquad \ldots\ldots \qquad (60)$$

$$P_N = \frac{1}{A'(0)} \left[h_1 + \frac{v_1}{u + v_2 + h_2} \left\{ h_2 + \frac{v_2 h_3}{u + v_3 + h_3} \right\} \right] m_h \qquad \ldots\ldots. \qquad (61)$$

$$P_i^m = \frac{u}{A'(0)} \left[1 + \frac{v_1}{u + v_2 + h_2} \left\{ 1 + \frac{v_2}{u + v_3 + h_3} \right\} \right] m_i \qquad \ldots\ldots. \qquad (62)$$

where $A'(0) = \left[\frac{d}{ds} A(s) \right]_{s=0}$ & $m_k = -\bar{S}'_k(0) \quad \forall \quad k = 1, 2, 3, h, i$

ANALYSIS OF CHARACTERISTICS

(A) The reliability of the system R(t) in terms of its Laplace transformation is

$$\bar{R}(s) = L.T[R(t)]$$

This can be obtained by assuming the failed states as absorbing i.e. repair rate to be zero.

$$\bar{R}(s) = \bar{P}_N^m(s) + \bar{P}_N^{m-1}(s) + \bar{P}_N^{m-2}(s) = \frac{1}{E(s)}[1 + B(s) + B(s).C(s)] \qquad \ldots\ldots \qquad (63)$$

on taking inverse Laplace Transform, one may get reliability of the system.

$$R(t) = \left[1 + \frac{v_1}{h_2 + v_2 - h_1 - v_1} + v_1 v_2 D \right] \text{Exp.}\{-(h_1 + v_1 + u)z\}$$

$$+ \left[v_1 v_2 E + \frac{v_1}{h_2 + v_2 - h_1 - u_1} \right] \text{Exp.}\{-(h_2 + v_2 + u)z\}$$

$$+ v_1 v_2 F \ \text{Exp.}\{-(h_3 + v_3 + u)z\} \qquad \ldots\ldots \qquad (64)$$

Where

$$D = \frac{1}{(h_2 + v_2 - h_1 - v_1)(h_3 + v_3 - h_1 - v_1)} \qquad \ldots\ldots \qquad (65)$$

$$E = -\frac{1}{(h_2 + v_2 - h_1 - v_1)(h_3 + v_3 - h_2 - v_2)} \qquad \ldots\ldots \qquad (66)$$

and

$$F = \frac{1}{(h_3 + v_3 - h_1 - v_1)(h_3 + v_3 - h_2 - v_2)} \qquad \ldots\ldots \qquad (67)$$

(B) The mean time to system failure is

$$\text{MTSF} = \lim_{S\to 0} \overline{R}(s)$$

$$\text{MTSF} = \left[1 + \frac{v_1}{h_2 + u_2 - h_1 - v_1} + v_1 v_2 D\right] \frac{1}{u + h_1 + v_1}$$

$$+ \left[v_2 v_1 E + \frac{v_1}{h_2 + v_2 - h_1 - v_1}\right] \frac{1}{u + h_2 + v_2} + \frac{v_1 v_2 F}{u + h_3 + v_3} \qquad \ldots\ldots \qquad (68)$$

Where D, E and F are mentioned in earlier equations. (65), (66) and (67) respectively.

NUMERICAL COMPUTATION

Let us consider the following values depending upon the significance of the components as $h_1 = 0.01, h_2=0.03, h_3=0.05, v_1=0.002, v_2=0.004$, $v_3 = 0.006$, $u = 0.05$ and $t = 0, 1, 2, 3\ldots$..then we get

$$D = \frac{1}{(0.022)(0.044)} = 1033.05785$$

$$E = \frac{1}{(0.022)(0.022)} = -2066.1157024$$

and $F = \frac{1}{(0.044)(0.022)} = 1033.05785$

Using these values in relation (64) and (63) we get

$$R(t) = (1.0991735)e^{-0.062t} + (-0.10743801)e^{-0.084t} + (0.0082644)e^{-0.106t} \qquad \ldots.\ \ldots\ldots \qquad (69)$$

The mean time to system failure with respect to u (failure rate of units of components of the system) is

$$\text{MTSF} = \lim_{s\to 0} R^*(S)$$

$$\therefore \text{MTSF} = (1.0991735).\frac{1}{u + h_1 + v_1} + (-0.10743801).\frac{1}{u + h_2 + v_2}$$

$$+ (0.0082644)\frac{1}{u + h_1 + v_2}$$

$$= \frac{1.00991735}{0.012 + u} - \frac{0.10743801}{0.034 + u} + \frac{0.0082644}{0.056 + u} \qquad \ldots\ldots \qquad (70)$$

Now for t = 0,1, 2, 3….. in equation (69), the reliability is shown in Figure 2.0

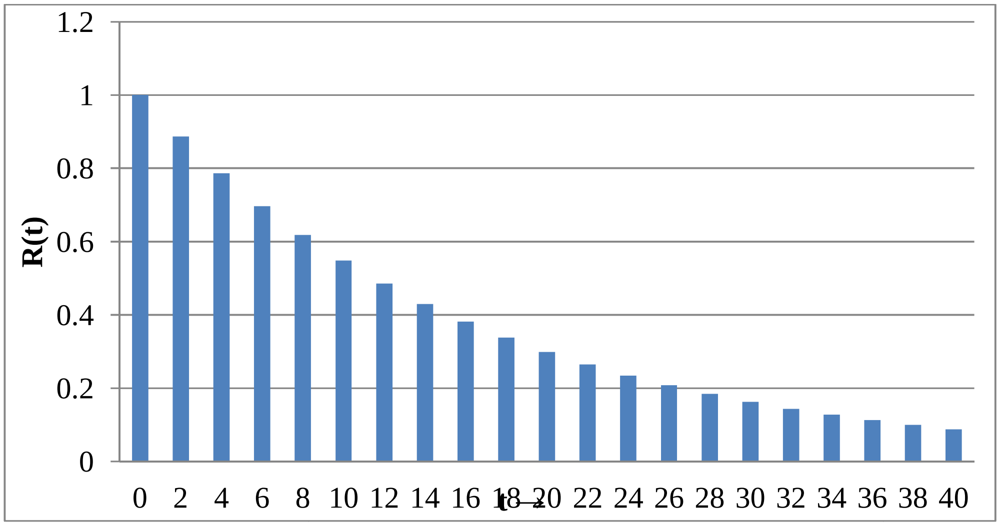

Figure 2.0

Now for u = 0, 1, 2, …..in Equation (70). The MTSF is shown in figure 3.0

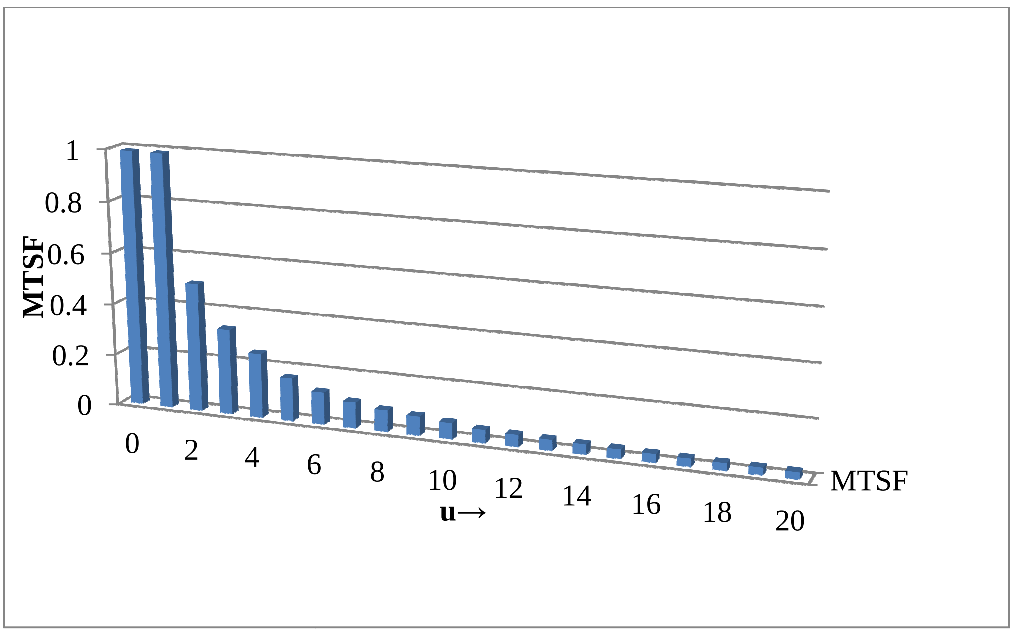

Figure 3.0

CONCLUSIONS

From the graph illustration of R(t) as shown in figure 2.0,It is concluded that as time increases the reliability(R(t)) of the system decreases rapidly and from the figure 3.0.

It is concluded that as the failure rate(u) increases,the value of MTSF decreases very smoothly and profit decreases whereas the values of u increases. So increment of any failure rate gives the smoothly decrement in the value of MTSF and profit function.

REFERENCES

1. Agnihotri, R.K. and S.K. Satsangi (1996) : "Two unit identical system with priority based repair and inspection", Microelectron. Reliab., 36, 279-282.
2. Billington, R. and R.M. Allan (1983) : "Reliability evaluation of Engineering System", Concepts and techniques, Plenum Press, New York.
3. Chung, W.K. (1991) : "Reliability analysis of a series system with repair", Microelectron. Reliab., 31, 363-365.
4. Dhanpal Singh,C.K. Goel and Ram Kishan, "Stochastic analysis of a two non- identical unit cold standby system where standby units stops functioning without failure", Journal of International Academy of Physical Science, Vol.(10) No.4(2006), pp. 91-101.
5. Dhanpal Singh,C.K.Goel and BaljeetKour, "Reliability Evaluation of series parallel system under head line repair policy" Published in Third National Conference (MATEIT),January 30-31,2010 in Convention Centre, University of Delhi, Delhi.
6. Gaver, D.P. (1963) : "Time to failure and availability of parallel systems with repair", IEEE Trans. Reliab., 12, 30-38.
7. Kishan, R., C.K. Goel and Dhanpal Singh (2004) : "Profit analysis of a complex system with correlated failures and repairs", Pure and Applied Mathematical Sciences, Vol. LX, No. 1-2.
8. Myres, R.H., K.L. wong and H.M. Gordy (1964) : "Reliability Engineering for Electronic Systems", John Wiley, New York.
9. Zang, Y.L. and T.P. Wand (1996) : "Repairable consecutive 2-out of System", Microelectron. Reliab., 36(5), 605-608.